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HDV-003-016302

Seat No. ______

M. Sc. (Mathematics) (Sem. III) (CBCS) Examination

November / December – 2017

MATH CMT - 3002 : Functional Analysis

(Old Course)

Faculty Code : 003
Subject Code : 016302

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Answer all questions.
(2) Each question carries 14 marks.
(3) The figures to the right indicate marks allotted to the question.

1 All are compulsory : (Each question carries 2 marks) **14**

(a) True or false? Justify Dual of a Hilbert space is a Hilbert space.

(b) Define equivalent norms on a n.l. space.

(c) True or false? Justify $\left(l^{\infty}, \|\cdot\|_{\infty}\right)$ has a Schauder basis.

(d) Give an example of a n.l. space which is not complete.

(e) Write the statement of Zorn's lemma and define sub linear functional.

(f) What is the meaning of the statement that the dual X' of X separates the points of X.

(g) If Y is closed subspace of a n.l. space X then give the definition of the induced norm on the quotient space X/Y.

2 Answer any **two** : **14**

(a) State and prove Minkowski's Inequality. **7**

(b) State without proof Open Mapping Theorem. Also prove that any two norms on a finite dimensional vector space X over K are equivalent. **7**

(c) Give an example to show that a metric on a vector space X need not be induced by a norm on X, with justification. **7**

3 All are compulsory : **14**

(a) State and prove Riesz lemma. **7**

(b) Prove that if every absolutely convergent series in $(X, \|\cdot\|)$ converges in $(X, \|\cdot\|)$ then $(X, \|\cdot\|)$ is a Banach Space over K. **7**

OR

3 All are compulsory : **14**

(a) State and prove closed graph theorem. **7**

(b) Define Canonical mapping C from a n.l. space X to X''. Prove that $C : X \to X''$ is an isometry. **7**

4 Answer any **two** : **14**

(a) State and prove Bessel's Inequality. **7**

(b) Give an example of an Inner Product Space which is not a Hilbert Space with justification. **7**

(c) State and prove the Hahn Banach spaces for n.l. Spaces over K. **7**

5 Answer any **two** : **14**

(a) Prove that a Banach space X over K cannot have a countably infinite hamel basis. **7**

(b) State and prove that characterization of finite dimensional n.l. spaces over K. **7**

(c) Let $X_1, X_2, \ldots\ldots., X_n$ be a n.l. space over K then $(X_1 \times X_2 \times \ldots\ldots\ldots. \times X_n, \|\cdot\|)$ is Banach space iff X_i is a Banach Space over K; for every $i = 1, 2, \ldots\ldots.., n$. Where $\|(x_1, x_2, \ldots\ldots\ldots., x_n)\| = \max \|x_i\|$ over i. **7**
